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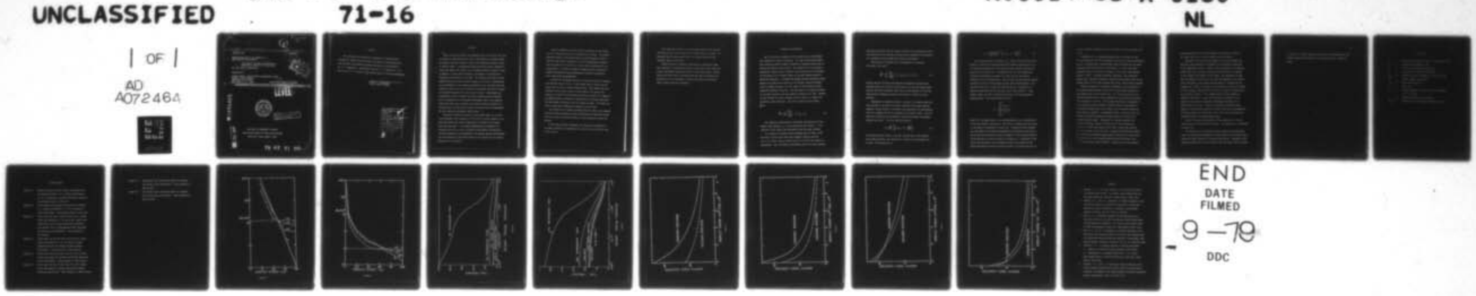

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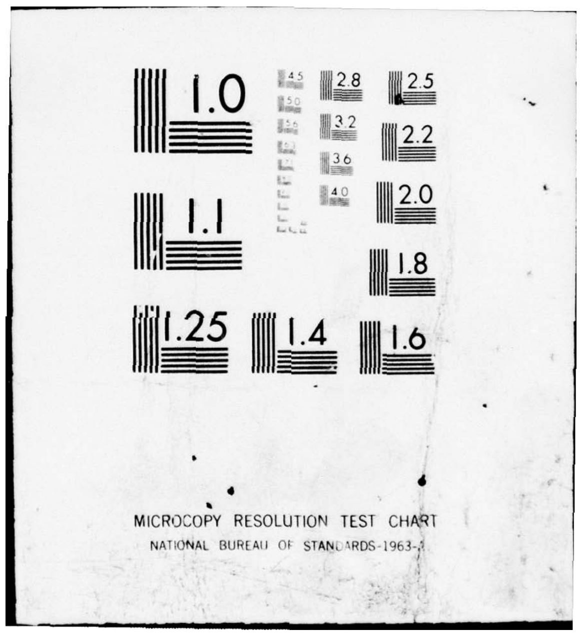
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6 COMPUTER ANALYSIS OF THE GROWTH OF A CLOUD DROP BY COALESCENCE.

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Final Report on Themis Sub-Project on Mathematical Modeling of Cloud Growth

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FOREWORD

The sub-project was funded by a Department of Defense Themis Contract No. N00014-68-A-0160, which was under the administration of the Institute of Atmospheric Sciences and under the supervision of Dr. R. C. Huntsinger. Contributors were Mr. T. E. Warborg, Mr. G. K. Bien and Mr. T. R. Nicholas, South Dakota School of Mines and Technology.

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Themis Project Manager

ABSTRACT

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Over a four year period, from September 1967 to July 1971, the rate of growth of a 30μ radius drop to 50 or 100μ radius by a process known as accretion has been studied as part of the numerical modeling effort of Project Themis. A mathematical model was derived which considers a single 30μ radius drop falling at its terminal velocity through a field of smaller, 5 to 20μ radius droplets, and growing by colliding and coalescing with droplets in its path. The droplets also fall at their terminal velocities and the rate of growth of the drop is proportional to the difference in the terminal velocities of the colliding drop and droplet as well as the area of sweep of the falling drop and the number of collectible droplets per unit volume of cloud. Hydrodynamic and electrostatic forces influence the rate of growth and are accounted for by introduction of a "collision efficiency" term to the analysis. Electrical forces also influence the terminal settling velocities of the particles, so Stoke's law of drag on a sphere and the intermediate law were derived incorporating these forces.

Warborg (1) studied the effect on drop growth rate of 0 to 3600 V/cm electrical field strengths, 10^{-18} to 10^{-13} coulomb charges on the drop and droplets and various populations of droplet sizes through which the drop falls. A 5μ radius monodispersion of droplets was considered as well as a pair of natural cloud droplet distributions. His study emphasizes the importance of the natural droplet distribution and the influence of electrical effects on the system as a 30μ radius drop grows to 50μ radius.

Bien (3) extended the growth range of the drop to 100μ radius using the unpublished collision efficiencies of Sartor. He investigated the effect on drop growth rate of field strengths of 298 to 2980 V/cm and droplet charges of 0 to $10r^2$ statcoulombs, assuming that the charge on the droplet is at all times proportional to its surface area. He studied a number of monodispersions and natural droplet distributions and approximated the droplet distribution with a mean-sized monodispersion.

Nicholas (5) considered the case of allowing the drop to collide with droplets ranging from 1 to 100μ radius some of which could be larger than as well as smaller than the drop. This reduces the time necessary for the drop to grow to 100μ radius by as much as 78%. Finally, the droplet distribution was allowed to evolve in a theoretical manner toward a situation in which each size category would have the same number of droplets per unit volume of cloud. The effect was again a significant reduction in necessary growth times.

The results of all workers indicate that the effect of an imposed electric field and charges on drops significantly increase the growth rate of the drop once minimum values of these electrical forces are reached.

In all cases studied, increasing the electrical influences in the model resulted in a decrease in the time for the 30μ drop to grow to 100μ radius.

Bien found that varying the liquid water content of the droplet distribution has a direct effect on the growth rate of the drop. He indicated that the drop growth rate is a linear function of the available water in the cloud.

Research efforts concerning modification of convective clouds have resulted in a mathematical model of cloud drop growth which can closely approximate conditions existing in an actual cloud. Modifications have been made to the model with the goal of improving the reliability of the evaluations of cloud drop growth by accretion under various electrical influences.

SUMMARY AND CONCLUSIONS

One of the major processes by which cloud water in transformed into rainwater is known as accretion. One type of accretion results from collisions between droplets subjected to external forces such as gravity and electric force fields. The probability of two droplets coalescing, combining upon collision, was the subject of the research of Sub-project III of Project Themis at the South Dakota School of Mines and Technology. A mathematical model has been developed investigating a 30μ radius drop falling at its terminal velocity through a field of smaller droplets (5 to 20μ radius), also falling at their respective terminal velocities. The drop grows by overtaking droplets in its path, colliding with and possibly combining with them. Warborg (1) has derived the growth equation which is based on a simple conservation of mass principle. The rate of change of drop radius, $\frac{dr}{dt}$ is:

$$\frac{dr}{dt} = \frac{\pi}{3r^2} \sum_{r_1=5}^{20} r_1{}^3 N_{r_1} A U_t \qquad (1)$$

The summation accumulates the individual contributions of each droplet size category, r_1, to the increase in drop radius, r. The remainder of the factors are explained in the following sections.

In a vacuum, the maximum cross-sectional area of sweep, A Cm^2, for which the drop which would just contact a droplet would be $\pi(r + r_1)^2$, where r and r_1 are the radii of the drop and droplet in centimeters. Upon introducing hydrodynamic forces and other external

electrical influences such as charged particles in an electrical field, the theoretical cross-section of sweep can be considered to be modified by a correction factor termed the "collision efficiency".

Modification of equation (1) to accommodate the collision efficiency term yields:

$$\frac{dr}{dt} = \frac{\pi}{3r^2} \sum_{r_1=5}^{20} r_1^3 N_{r_1} E_c (r + r_1)^2 U_t \tag{2}$$

Warborg used the collision efficiencies calculated by Plumlee which provided values for collision between drops less than 50μ radius under various electrical influences while Bien (3) and Nicholas (5) used the theoretically determined collision efficiencies of Sartor in their analyses.

Computation of terminal velocity, U_t cm/sec, for falling drops has been obtained by modifying the Stokes' equation for a sphere falling through a viscous medium, to include a term accounting for electrostatic force on a charged drop in the presence of an electric field. Bien (3) shows this derivation in his master's thesis which holds for Reynolds numbers less than 2. The final equation becomes:

$$U_t = \frac{2r^2}{9\mu} \left[g (\rho_p - \rho) - \frac{3qE}{4r^3} \right] \tag{3}$$

For Reynolds numbers between 2 and 500, another form of the equation from McCabe and Smith, see Nicholas (5), called the intermediate law, is used. The formulation is:

$$U_t = \frac{.153\ (2r)^{1.14}}{\mu^{.43}\ \rho^{.28}} \left[g\ (\rho_p - \rho) - \frac{3qE}{4r^3} \right] \quad (4)$$

For the distribution of droplets, N_{r_1} through which the drop falls, Warborg (1) used a 10μ monodispersion and a natural dispersion of 5 to 20μ droplets determined for fair weather cumulus clouds by Battan and Reitan shown in Figure (1). He indicated that the full droplet distribution significantly increases the growth rate of the drop as compared with the monodispersion. Bien (3) used the same natural droplet distribution and also investigated various other monodispersions. His analysis shows that a droplet distribution can be approximated by a mean-size monodispersion with the same liquid water content. A mean droplet radius was used to determine the size of the representative monodispersion. The equation takes the form:

$$\bar{r}_1 = \frac{\sum_{r_1} N_{r_1}\ r_1^{\ 5}}{\sum_{r_1} N_{r_1}\ r_1^{\ 4}}$$

Where $\bar{r}_1$ is the mean radius of the monodispersion in cm, representative of the full droplet distribution. The effect of applying this principle to the model is illustrated in Figure (3). Nicholas (5) then extended the size range of the droplet distribution by introducing a new distribution which was presented by Junge including 1 to 100μ radius droplets. The distribution is shown as curve tl in Figure (2). Comparison of the growth rate produced by this distribution with that produced by the Battan and Reitan distribution shows the effect of allowing the drop to

grow by collecting droplets which may be larger as well as smaller than itself.

Comparison of the constant spectrum curves of Figures (5) and (7) and Figures (6) and (8) illustrates that using the Junge distribution results in substantially reduced growth times as compared to the cases involving the Battan and Reitan distribution. At small electrical parameter values the percent of decrease in the growth times reached as high as 200%. At larger electrical influences, this difference becomes less pronounced reaching a minimum of approximately 100% at the maximum values of the field strength and surface charge density studied.

Finally, Nicholas (5) developed a theoretical time evolution of the droplet spectrum based on the limit of equal number density for each droplet size at some future time. The time evolutions for the Battan and Reitan and the Junge distributions are depicted by Nicholas (5) as Figures (1) and (2). The original distribution, indicated by t1 is allowed to evolve toward a limiting condition represented by time-curve t4 wherein all of the droplet size categories contain an equal number of droplets. Intermediate values, curves t2 and t3 are obtained by linear interpolation between points on t1 and t4 for constant droplet radius. Comparison of "constant spectrum" and "evolving spectrum" curves in Figures (5) to (8) indicate that substantial growth time savings (22 to 78%), result from allowing the distributions to age with time. Inconsistent portions of Figures (7) and (8) at low surface charge density resulting in a lower value for growth time of the constant spectrum have been traced to the collision efficiency data of Sartor. Incorporation of the evolving

droplet spectrum into the model produced a significant increase in the growth rate of the drop for almost every case tested.

Electrical effects have been central to all three analyses and have been used throughout as parameters of the computer simulations of the mathematical models. Warborg, Bien, and Nicholas all depicted growth times for a specific electrical field strength as a function of the charge of the droplet of the system. The assumption is made that the charge on a drop is at all times proportional to the surface area of the drop, or the surface charge density is constant for one computer run. Hence, the value in the abscissa of Figures (5) through (8) for instance, multiplied by $4\pi r^2$ gives the charge in statcoulombs on the drop or radius r. The constant electric field strength is marked on each graph. In all cases observed, an increase in the electric field strength produced a corresponding increase in the growth rate of the drop. Likewise, increasing the surface charge density on the droplets increases the drop growth rate correspondingly. This trend agrees with other work in the field and substantiates the assumptions made in the mathematical model.

Bien's (3) analysis determined a linear dependence of the drop growth rate on cloud liquid water content. The dependence is illustrated in Figure (4).

The direction of the research efforts regarding the mathematical model has been toward formulation of a more realistic drop growth model, including effects observed in an actual cloud parcel. Knowledge of the growth characteristics of cloud droplets and the factors which influence

the growth can provide valuable insight into the understanding of the overall precipitation mechanism in the modification of convective clouds.

NOMENCLATURE

A	=	Cross-sectional area of sweep of a falling drop, cm^2.
E	=	Electric field strength, V/cm.
E_c	=	Collision efficiency, dimensionless.
g	=	Acceleration of gravity, 981 cm/sec^2.
N_{r_1}	=	Number of droplets of radius r_1 per cm^3 of cloud.
q	=	Droplet charge, statcoulomb.
r, r_1	=	Drop, droplet radii, cm.
t	=	Time, sec.
U_i	=	Approach velocity of large drop to small droplet, $(U_L - U_s)$ cm/sec.
μ	=	Dynamic viscosity of air, g/cm/sec.
ρ, ρ_p	=	Densities of air and water respectively, g/cm^3.

FIGURE LEGENDS

Figure (1). Battan and Reitan discrete droplet distribution and theoretical evolution. t1 = original distribution, t2 → t4 = evolution to constant distribution function. Liquid water content = 0.665 g/m^3.

Figure (2). Junge droplet distribution and theoretical evolution. t1 = original distribution, t2 → t4 = evolution to final distribution. Liquid water content = 0.421 g/m^3.

Figure (3). Growth times for drop to grow from 30 to 100 μ radius versus drop charge for 5, 10, and 18.32 μ radius monodispersions for the Battan and Reitan distribution. Curve marked "18.32 μ Monodispersed Cloud" represents calculated mean monodispersion. Field strength E = 298 volts/cm.

Figure (4). Growth times for drop to grow from 30 to 100 μ radius versus drop charge for 5, 10, 15, and 20 μ radius monodispersions for the Battan and Reitan droplet distribution. Field strength E = 2980 volts/cm.

Figure (5). Growth times for drop to grow from 30 to 100 μ radius versus drop charge for constant and evolving Battan and Reitan distribution. Field strength E = 298 volts/cm.

Figure (6). Growth times for drop to grow from 30 to 100 μ radius versus drop charge for constant and evolving Battan and Reitan distribution. Field strength E = 2980 volts/cm.

Figure (7). Drop growth times versus drop charge for constant and evolving Junge distribution. Field strength E = 298 volts/cm.

Figure (8). Drop growth times versus drop charge for constant and evolving Junge distribution. Field strength E = 2980 volts/cm.

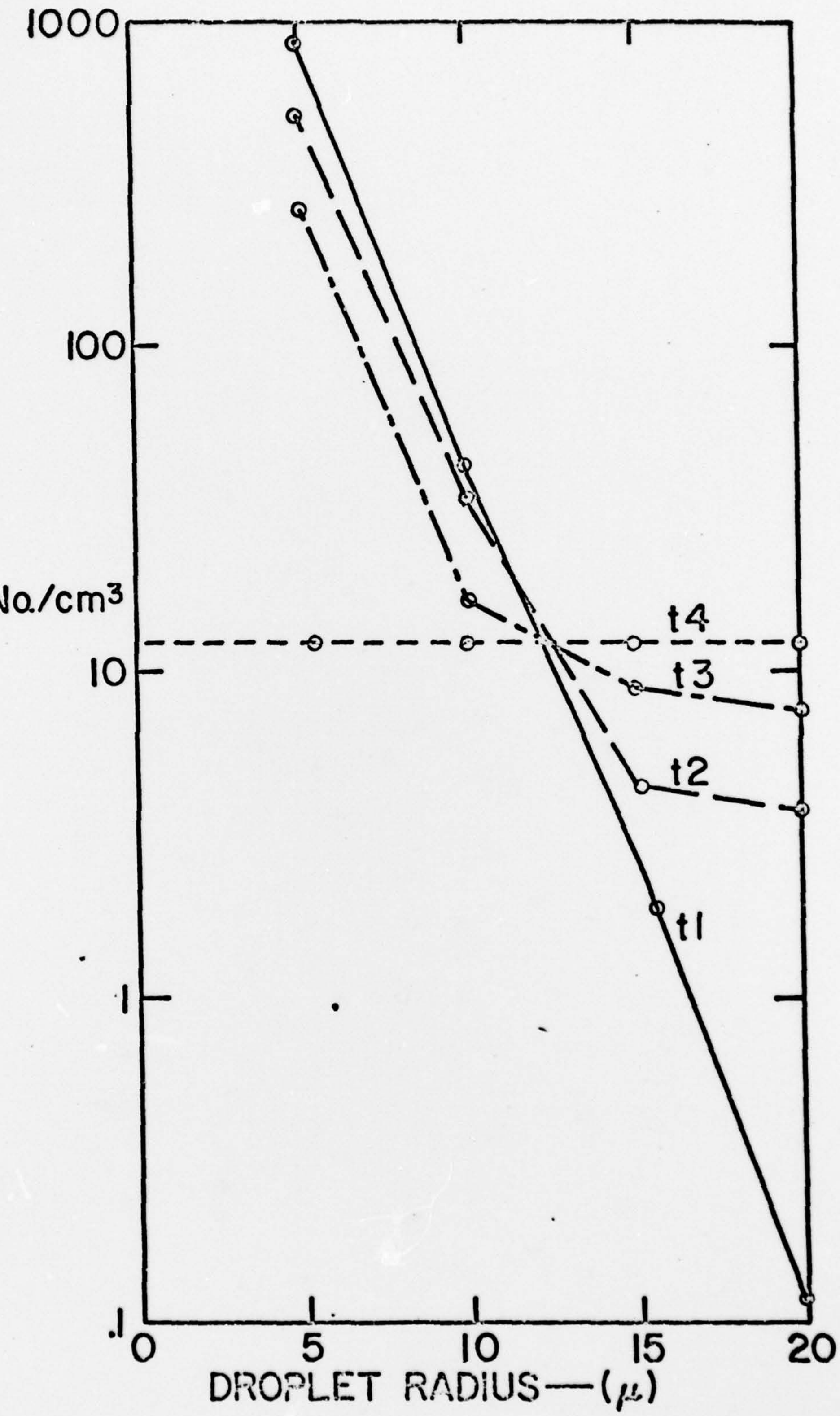


Figure 1.

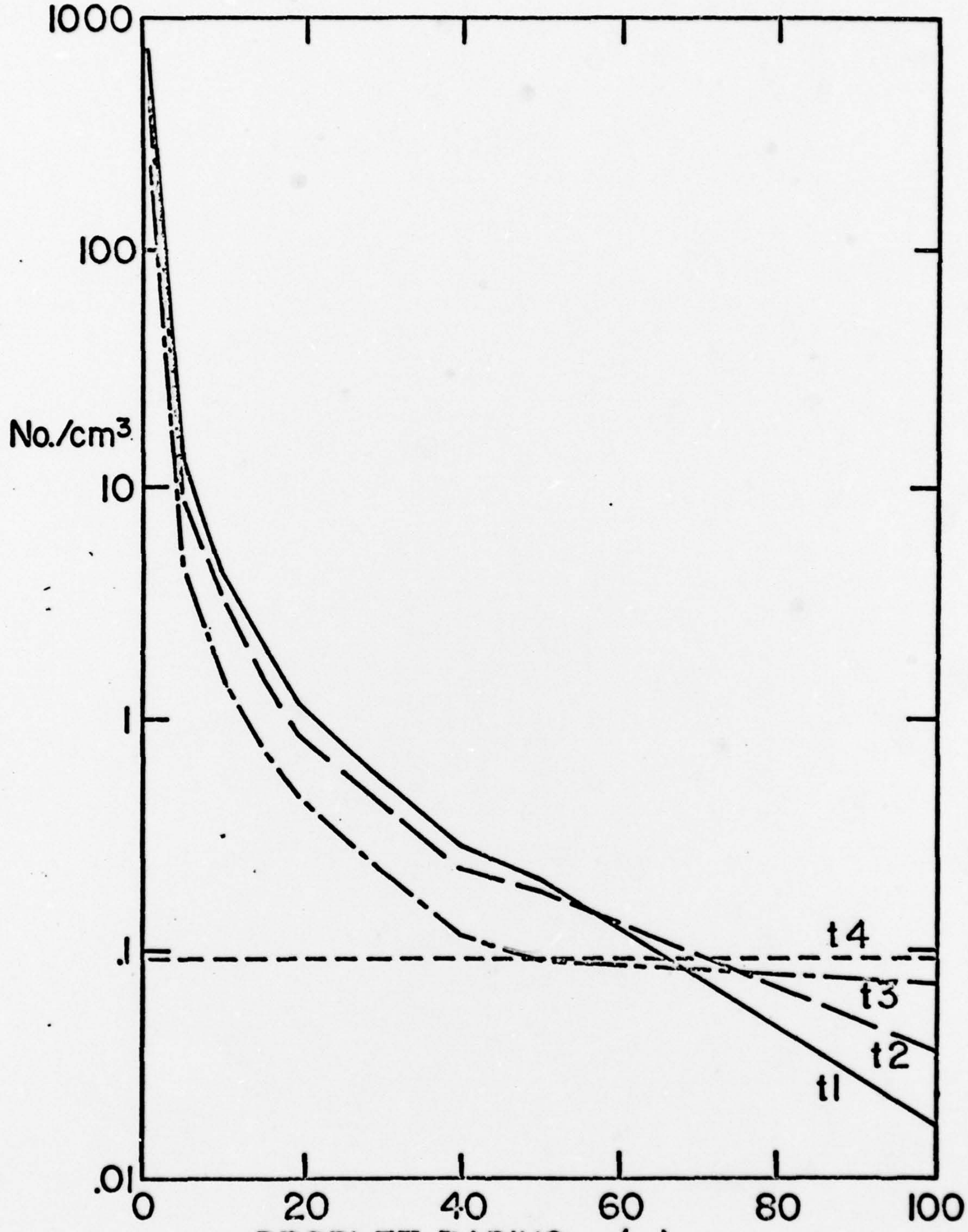
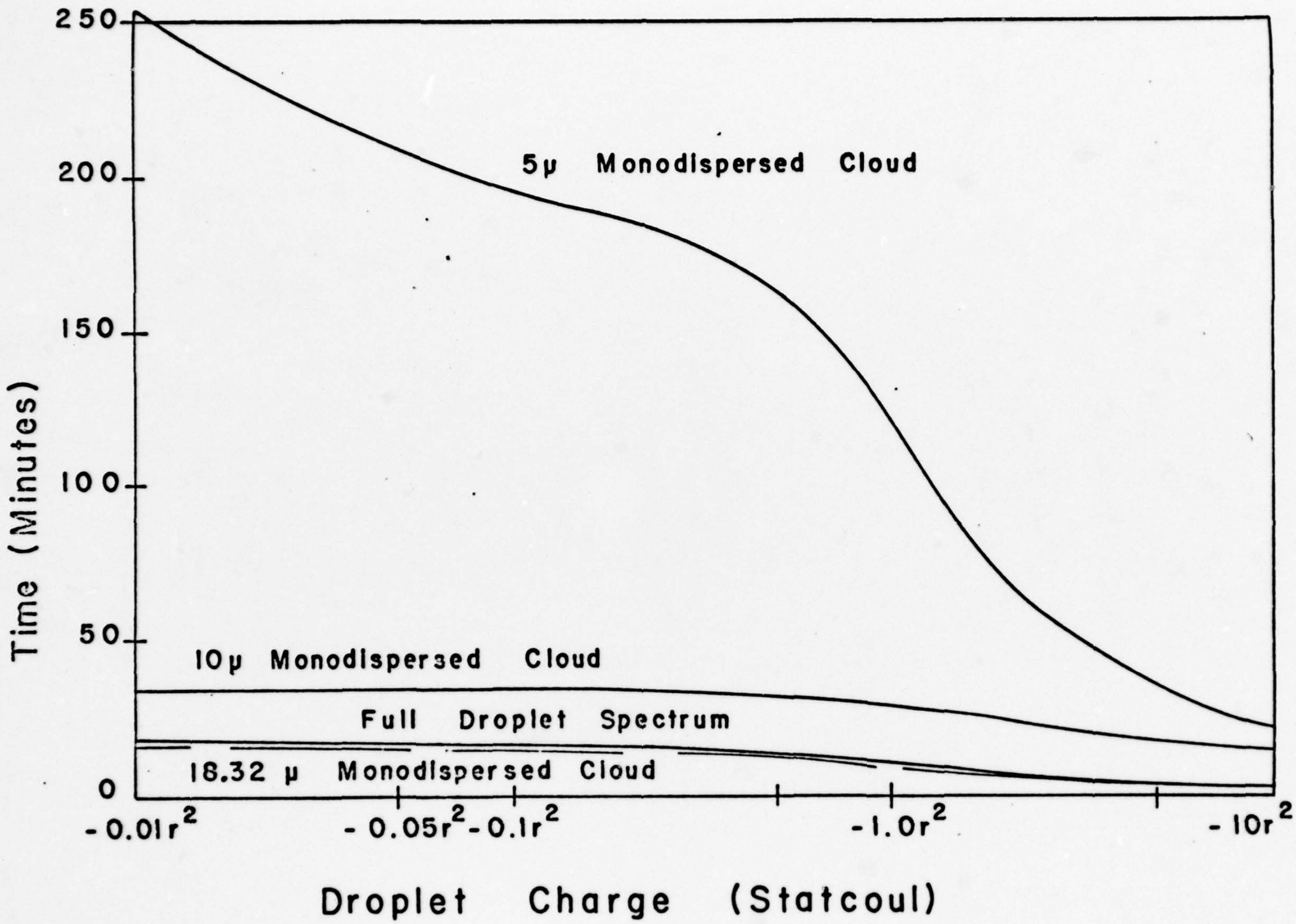

1000
100
No./cm3
10
1
.1
.01
0
20
40
60
80
100
DROPLET RADIUS —(μ)
t4
t3
t2
t1

Figure 2.



Time (Minutes)
250
200
150
100
50
0
5μ Monodispersed Cloud
10μ Monodispersed Cloud
Full Droplet Spectrum
18.32 μ Monodispersed Cloud
-0.01r²
-0.05r²
-0.1r²
-1.0r²
-10r²
Droplet Charge (Statcoul)

Figure 3.

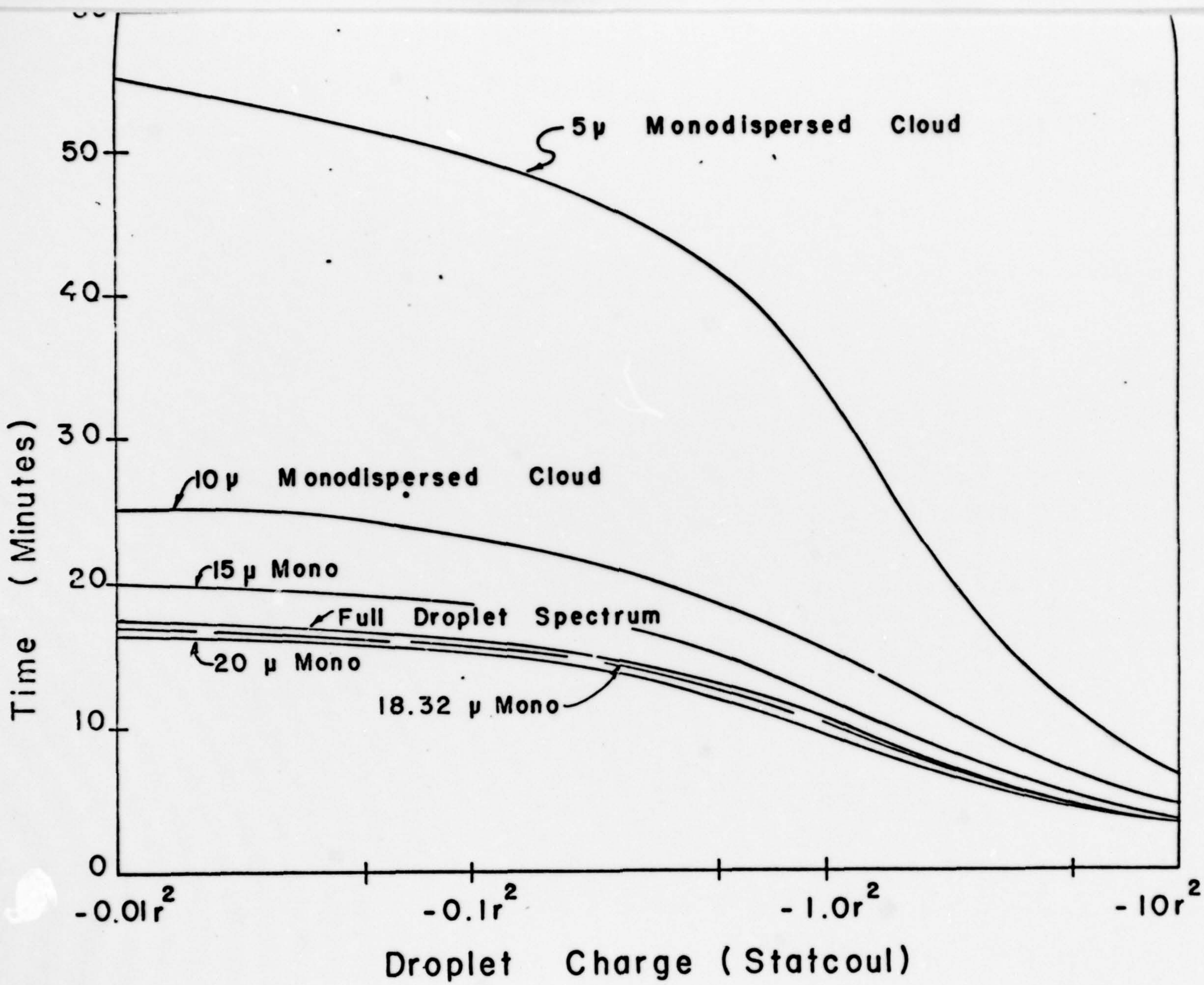

5 μ Monodispersed Cloud
10 μ Monodispersed Cloud
15 μ Mono
Full Droplet Spectrum
20 μ Mono
18.32 μ Mono
Time (Minutes)
50
40
30
20
10
0
-0.01r²
-0.1r²
-1.0r²
-10r²
Droplet Charge (Statcoul)

Figure 4.

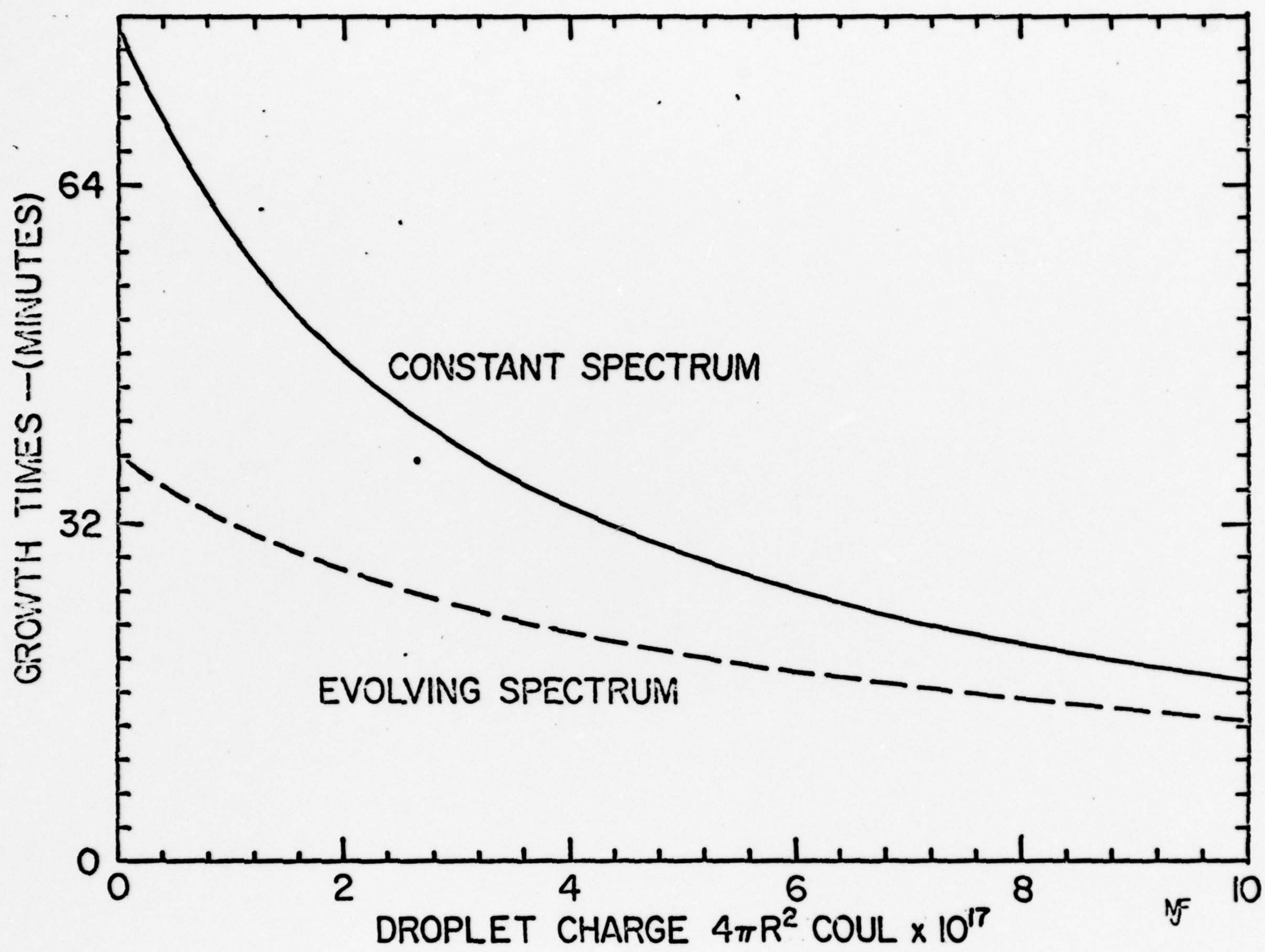

GROWTH TIMES —(MINUTES)
64
32
0
CONSTANT SPECTRUM
EVOLVING SPECTRUM
0
2
4
6
8
10
DROPLET CHARGE 4πR² COUL x 10¹⁷

Figure 5.

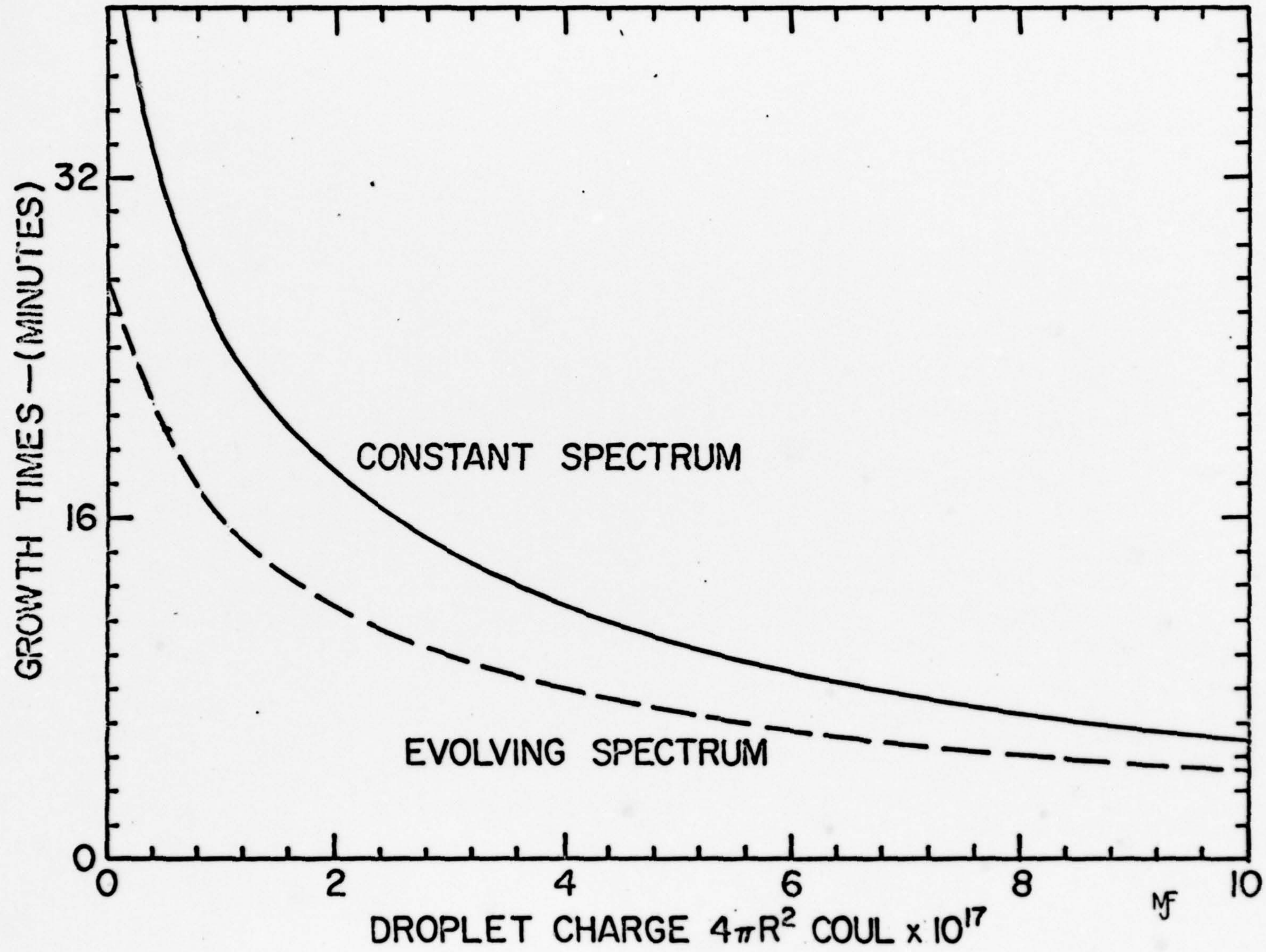

GROWTH TIMES —(MINUTES)
32
16
0
CONSTANT SPECTRUM
EVOLVING SPECTRUM
0
2
4
6
8
10
DROPLET CHARGE 4πR² COUL x 10¹⁷
MF

Figure 6.

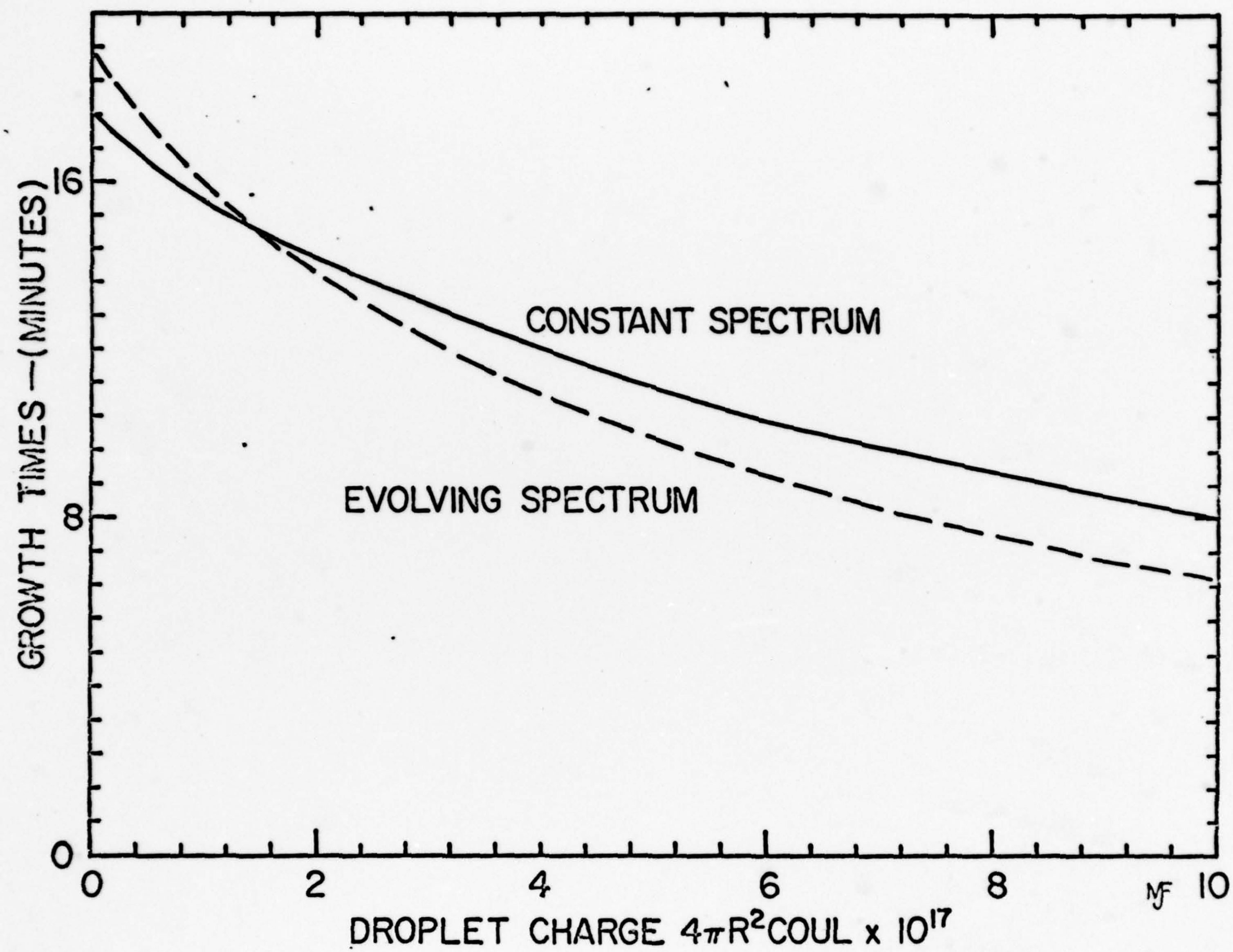

GROWTH TIMES —(MINUTES)
16
8
0
CONSTANT SPECTRUM
EVOLVING SPECTRUM
0
2
4
6
8
10
DROPLET CHARGE 4πR²COUL x 10¹⁷
MF

Figure 7.

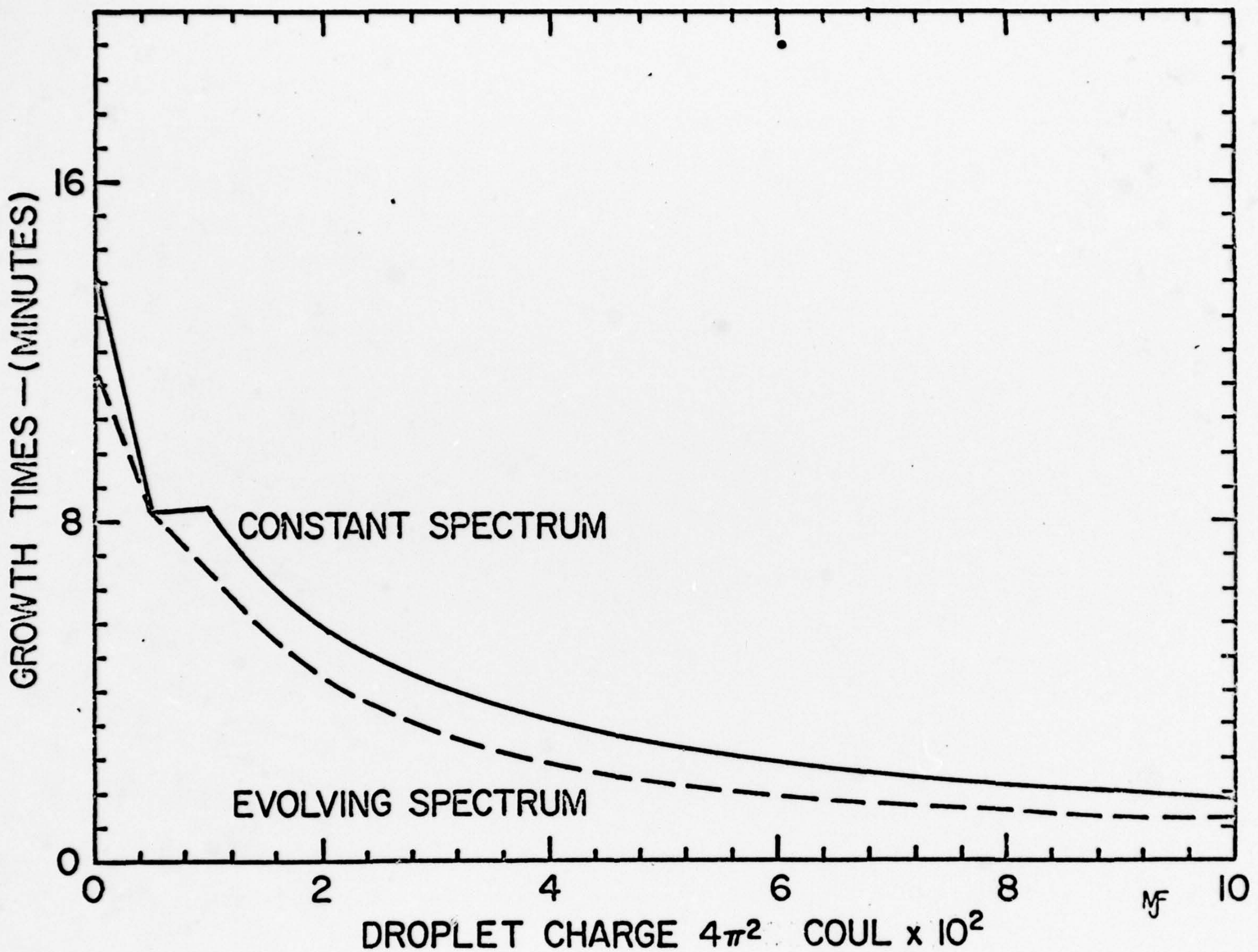

GROWTH TIMES —(MINUTES)
16
8
0
CONSTANT SPECTRUM
EVOLVING SPECTRUM
0
2
4
6
8
10
DROPLET CHARGE 4π2 COUL x 10²
MJF

Figure 8.

CONTENTS

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